

Conventional Game Theory



John Nash

Conventional Game Theory

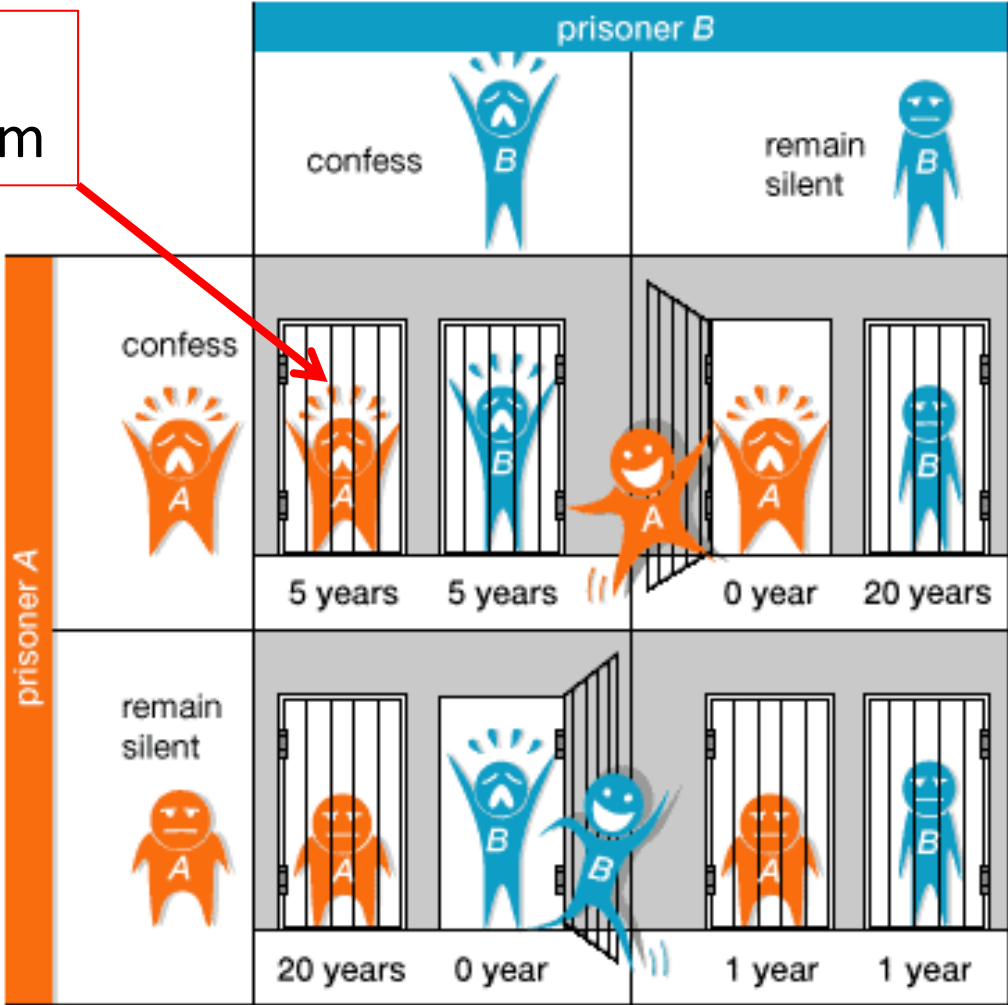
- Nash Equilibrium



- Nash, John (1950) "Equilibrium points in n-person games" [*Proceedings of the National Academy of Sciences* 36\(1\):48-49.](#)
- Nash, John (1951) "Non-Cooperative Games" [*The Annals of Mathematics* 54\(2\):286-295.](#)

Prisoner's Dilemma Game

Nash Equilibrium



Picture from Encyclopedia Britannica, 2006

Equilibrium Concepts

- **Nash Equilibrium** - every strategy in any player's strategy profile is a **best response** to every other strategy in the profile.
- In other words, there is no strategy that a player could play that would yield a higher payoff, given all the strategies played by the other players (**remember the BLOND!!!**).
- What does it mean? **People are rational, they anticipate what others will do and they do not do anything stupid (given what others do).**

Equilibrium Concepts

- **Bayesian Nash Equilibrium** - is defined as a strategy profile and beliefs specified for each player about the types of the other players that **maximizes the expected payoff** for each player **given their beliefs** about the other players' types and given the strategies played by the other players.
- What does it mean? **People are rational, they have beliefs about other player's types and they anticipate what other players will do and they do not do anything stupid (given their beliefs about others' types and their predictions of others' actions).**

Quantal Response Equilibrium



Richard McKelvey



Thomas Palfrey

Equilibrium Concepts

- **Quantal Response Equilibrium** – best response functions are stochastic: people are more likely to choose actions with higher expected payoff, but best responses are not chosen with 100% probability.
- What does it mean? **QRE allows every strategy to be played with non-zero probability, and so any action is possible (though not necessarily reasonable). In other words, people may do something stupid.**

Symmetric Matching Pennies



goalie
vs
kicker
problem

Gianluigi Buffon



Asymmetric Matching Pennies

- [Goeree, J., Holt, Ch. and T. Palfrey \(2000\)](#) provide an example of two asymmetric matching pennies games

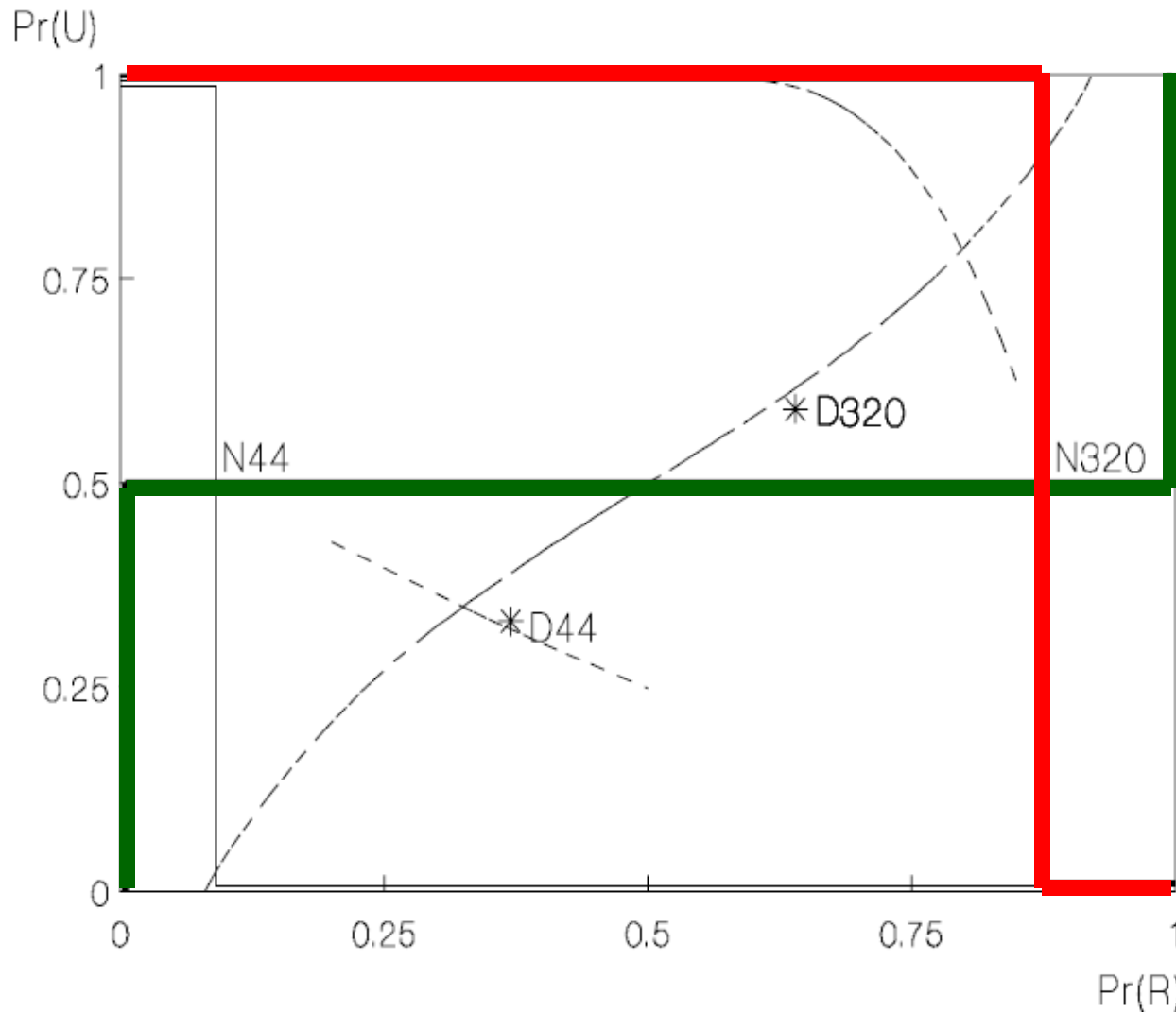
		COLUMN PLAYER	
		L	R
ROW PLAYER	U	X 40	80
	D	40	X 80

Game 1: $X=320$ Game 2: $X=44$

Notice that if $X=80$, the probability of playing each decision for both players is 0.5 (or 50%)

Asymmetric Matching Pennies

If $X > 80$ or
in Game 1
 $X = 320$



Unique NE
at N320

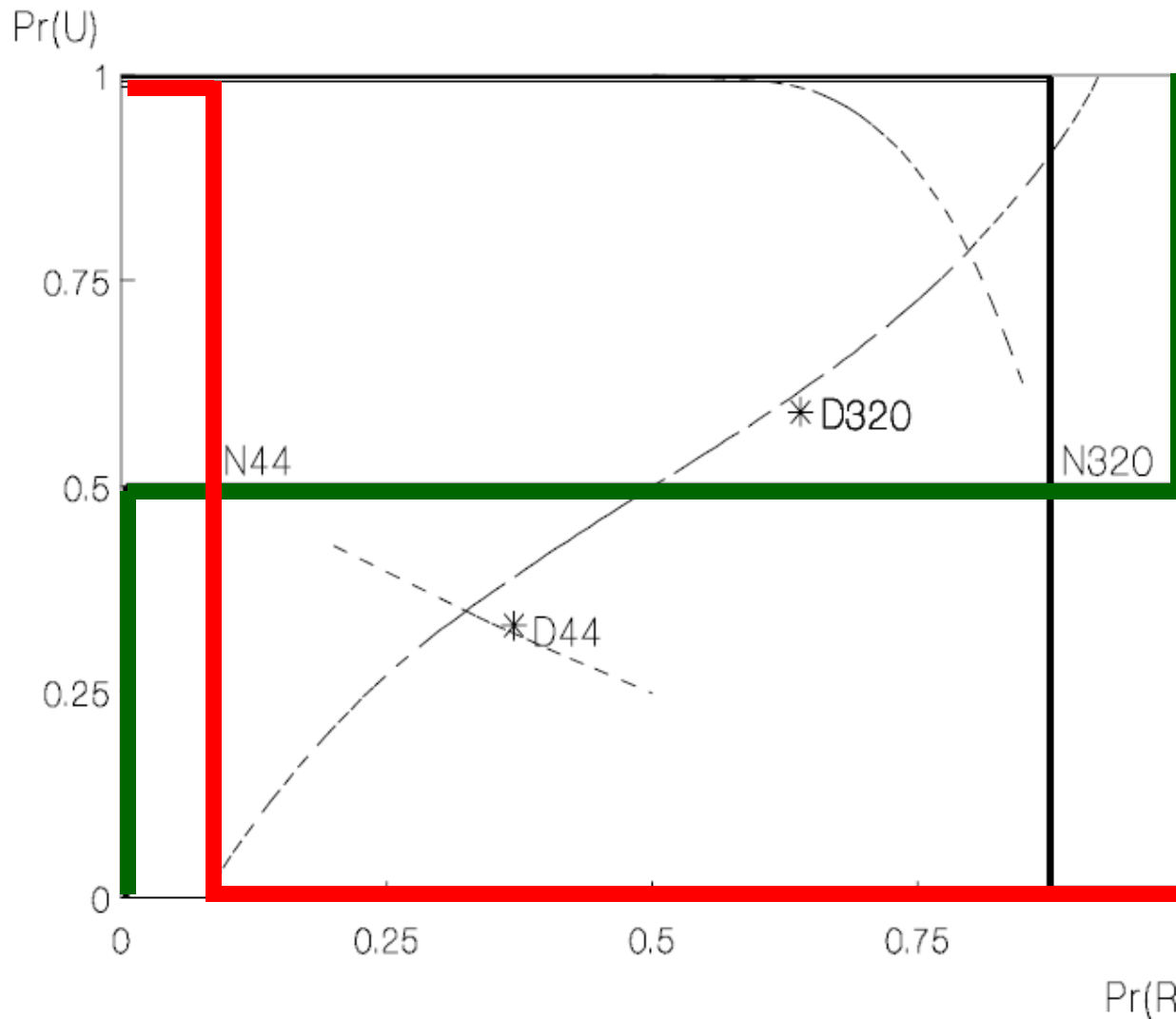
$p(R) = 0.875$

$p(U) = 0.5$

Example from [Goeree, J., Holt, Ch. and T. Palfrey \(2000\)](#)

Asymmetric Matching Pennies

If $X < 80$ or
in Game 2
 $X = 44$



Unique NE
at N44

$p(R) = 0.1$

$p(U) = 0.5$

Example from [Goeree, J., Holt, Ch. and T. Palfrey \(2000\)](#)

QRE in Asymmetric Matching Pennies

- We can calculate (logit) quantal response equilibrium for both games:

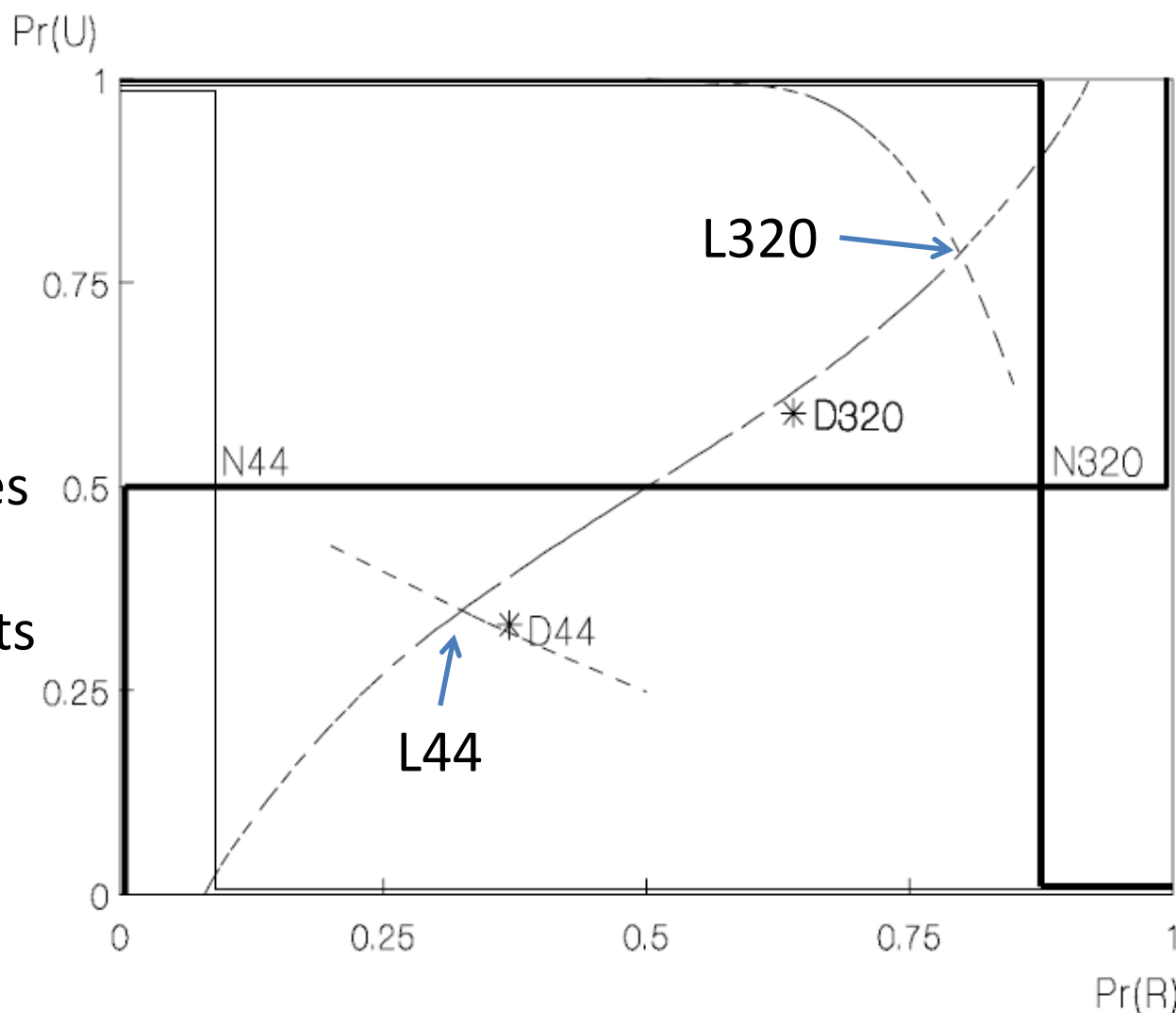
$$P_R = \frac{\exp(\pi^e(\mathbf{R})/\mu)}{\exp(\pi^e(\mathbf{L})/\mu) + \exp(\pi^e(\mathbf{R})/\mu)},$$

$$P_U = \frac{\exp(\pi^e(\mathbf{U})/\mu)}{\exp(\pi^e(\mathbf{D})/\mu) + \exp(\pi^e(\mathbf{U})/\mu)}.$$

Example from [Goeree, J., Holt, Ch. and T. Palfrey \(2000\)](#)

QRE in Asymmetric Matching Pennies

D320 and D44 are aggregate choice percentages from experiments



Example from [Goeree, J., Holt, Ch. and T. Palfrey \(2000\)](#)

QRE in Asymmetric Matching Pennies

Bottom line of the example we have just considered:

- 1) QRE alone cannot explain the data (WHY?)
- 2) Risk aversion alone cannot explain the data (WHY?)
- 3) We observe that the high payoff $X=320$ in Game 1 is undervalued by the participants in experimental studies
- 4) Possible explanation? The data could be explained only when risk aversion and QRE are taken together.

Example from [Goeree, J., Holt, Ch. and T. Palfrey \(2000\)](#)