

## MODELS OF STOCHASTIC CHOICE AND DECISION THEORIES: WHY BOTH ARE IMPORTANT FOR ANALYZING DECISIONS

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### SUMMARY

We select a menu of seven popular decision theories and embed each theory in five models of stochastic choice, including tremble, Fechner and random utility model. We find that the estimated parameters of decision theories differ significantly when theories are combined with different models. Depending on the selected model of stochastic choice we obtain different rankings of decision theories with regard to their goodness of fit to the data. The fit of all analyzed decision theories improves significantly when they are embedded in a Fechner model of heteroscedastic truncated errors or a random utility model. Copyright © 2009 John Wiley & Sons, Ltd.

*Received 15 January 2008; Revised 17 December 2008*

### 1. INTRODUCTION

Experimental studies on repeated decision making under risk find that individuals often do not choose the same alternative when they are faced with an identical binary choice problem replicated within a short period of time. For instance, Camerer (1989) reports that 31.6% of subjects reversed their initial decision on the second repetition of a choice task. Starmer and Sugden (1989) observe a switching rate of 26.5%. Hey and Orme (1994) find that 25% of repeated decisions were inconsistent, even when individuals were allowed to declare indifference. Ballinger and Wilcox (1997) report a median switching rate of 20.8%.

An overwhelming majority of decision theories are deterministic; i.e., they predict that an individual makes identical choices if a decision problem is repeated, unless she is exactly indifferent. Such a decision theory is typically embedded into a model of stochastic choice, when there is a need to relate a deterministic theory to stochastic data (e.g., Regenwetter *et al.*, 1999). Thus a model of stochastic choice serves as an intermediary, which translates a deterministic prediction of decision theories into a stochastic choice pattern that can be estimated by econometric methods on empirical data.

Much of empirical research on decision making under risk focuses on comparing numerous decision theories according to their goodness of fit to the behavioral patterns observed in the laboratory experiments. A researcher typically selects one model of stochastic choice and estimates several decision theories all embedded into this model. For example, Harless and Camerer (1994) estimate decision theories embedded in a tremble model. Hey and Orme (1994) estimate decision

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theories embedded in a Fechner model of homoscedastic random errors. A less common procedure is to select one or two decision theories and to estimate them in conjunction with several models of stochastic choice (e.g. Carbone, 1997; Loomes *et al.*, 2002). As Hey (2005) points out, the selection of a model of stochastic choice is generally considered to be of secondary importance, with far more emphasis put on the selection of a decision theory. However, Buschena and Zilberman (2000) and Blavatskyy (2007) show that conclusions drawn from the estimation of decision theories embedded in one model of stochastic choice do not necessarily hold if theories are estimated with different models of stochastic choice.

In this paper we do not estimate several decision theories all embedded in one model of stochastic choice or just one decision theory embedded in several models of stochastic choice. To look at the big picture, we estimate seven popular decision theories, each embedded in five models of stochastic choice that were proposed in the literature. In addition, we do not run a conventional laboratory experiment. Instead, we use data from a natural experiment, where representatives of a large fraction of the adult population choose between a risky lottery with large monetary stakes and an amount for certain.

Our main findings can be summarized as follows. Estimates of decision theories vary significantly when theories are embedded in different models of stochastic choice. Two models that provide the best description of the data are the Fechner model of heteroscedastic truncated errors and a random utility model. Decision theory with the best fit to the data depends on the selected model of stochastic choice. Across all models that we consider in this paper, the best explanation of the data is provided by either expected utility theory with expo-power utility function or regret theory.

The remainder of this paper is organized as follows. Models of stochastic choice are described in Section 1. Section 2 presents the data from a natural experiment. Section 3 describes the estimation procedure. Section 4 compares the estimates of seven decision theories embedded in five models of stochastic choice. Section 5 concludes.

## 2. MODELS OF STOCHASTIC CHOICE

### 2.1. Tremble model

Harless and Camerer (1994) argue that individuals generally choose among lotteries according to a deterministic decision theory, but there is a constant probability that this deterministic choice pattern reverses (as a result of pure tremble). Let  $\theta$  be a vector of parameters that characterize the parametric form of a decision theory and let  $u(L, \theta)$  denote utility of a lottery  $L$  according to this theory. In this paper we consider only binary choices between a risky lottery and a degenerate lottery that delivers one monetary outcome with probability one. According to Harless and Camerer (1994), the log-likelihood of observing  $N$  decisions when individuals choose a risky lottery  $L_i$  over a monetary amount  $O_i$  for certain, for all  $i \in \{1, \dots, N\}$ , can be written as

$$\begin{aligned} \text{LL}_R = & \sum_{i=1}^N \log(1-p) \cdot I(u(L_i, \theta) > u(O_i, \theta)) + \sum_{i=1}^N \log(p) \cdot I(u(L_i, \theta) < u(O_i, \theta)) \\ & + \sum_{i=1}^N \log(1/2) \cdot I(u(L_i, \theta) = u(O_i, \theta)) \end{aligned} \quad (1)$$

and the log-likelihood of observing  $M$  decisions when individuals choose a monetary amount  $O_i$  for certain over a risky lottery  $L_i$ , for all  $i \in \{1, \dots, N\}$ , can be written as

$$\begin{aligned} \text{LL}_A = & \sum_{i=1}^M \log(p) \cdot I(u(L_i, \theta) > u(O_i, \theta)) + \sum_{i=1}^N \log(1-p) \cdot I(u(L_i, \theta) < u(O_i, \theta)) \\ & + \sum_{i=1}^N \log(1/2) \cdot I(u(L_i, \theta) = u(O_i, \theta)) \end{aligned} \quad (2)$$

where  $I(x)$  is an indicator function, i.e.  $I(x) = 1$  if  $x$  is true and  $I(x) = 0$  if  $x$  is false, and  $p \in (0, 1)$  is probability of a tremble. Note that a tremble occurs when the utility of a lottery is less than the utility of a sure amount but an individual chooses the lottery; or when the risky lottery yields a higher utility but an individual chooses the sure amount. Parameters  $\theta$  and  $p$  are estimated to maximize combined log-likelihood  $\text{LL}_R + \text{LL}_A$ .

## 2.2. Fechner model of homoscedastic random errors

Hey and Orme (1994) estimate a Fechner model of random errors, where a random error distorts the net advantage of one lottery over another (in terms of utility). Net advantage is calculated according to an underlying deterministic decision theory. The error term is a normally distributed random variable with zero mean and constant standard deviation. According to Hey and Orme (1994), the log-likelihood of observing  $N$  decisions when individuals choose a risky lottery  $L_i$  over a monetary amount  $O_i$  for certain, for all  $i \in \{1, \dots, N\}$ , can be written as

$$\text{LL}_R = \sum_{i=1}^N \log(\Phi_{0,\sigma}[u(L_i, \theta) - u(O_i, \theta)]) \quad (3)$$

and the log-likelihood of observing  $M$  decisions when individuals choose a monetary amount  $O_i$  for certain over a risky lottery  $L_i$ , for all  $i \in \{1, \dots, M\}$ , can be written as

$$\text{LL}_A = \sum_{i=1}^M \log(1 - \Phi_{0,\sigma}[u(L_i, \theta) - u(O_i, \theta)]) \quad (4)$$

where  $\Phi_{0,\sigma}[\cdot]$  is the cumulative distribution function (cdf) of a normal distribution with zero mean and standard deviation  $\sigma$ . Parameters  $\theta$  and  $\sigma$  are estimated to maximize combined log-likelihood  $\text{LL}_R + \text{LL}_A$ . Note that a revealed choice decision is a binary dependent variable (an individual either accepts or rejects a sure amount) and we cannot simply use ordinary least squares estimation. Instead, a probit or logit latent variable model is more appropriate. The Fechner model of homoskedastic random errors described above is equivalent to a standard probit model.

## 2.3. Fechner model of heteroscedastic random errors

Hey (1995) and Buschena and Zilberman (2000) extend a Fechner model of random errors by assuming that the error term is heteroscedastic; i.e., the standard deviation of errors is higher in

certain decision problems, for example, when lotteries have many possible outcomes. Blavatskyy (2007) argues that individuals who face risky lotteries with a smaller range of possible outcomes have a lower volatility of random errors than individuals who face risky lotteries with a wider range of possible outcomes. For example, an individual facing a 50 : 50 chance of €1 and €5 is likely to have a smaller variance of random errors than an individual facing a 50 : 50 chance of €1 and €500,000. We will estimate a Fechner model of random errors when the standard deviation of random errors is proportionate to the difference between the utility of the highest outcome  $\bar{x}$  and the utility of the lowest outcome  $\underline{x}$  of a risky lottery  $L$ . Specifically, the log-likelihood of observing  $N$  decisions when individuals choose a risky lottery  $L_i$  over a monetary amount  $O_i$  for certain, for all  $i \in \{1, \dots, N\}$ , can be written as

$$LL_R = \sum_{i=1}^N \log \left( \Phi_{0,\sigma} [u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)] [u(L_i, \theta) - u(O_i, \theta)] \right) \quad (5)$$

and the log-likelihood of observing  $M$  decisions when individuals choose a monetary amount  $O_i$  for certain over a risky lottery  $L_i$ , for all  $i \in \{1, \dots, M\}$ , can be written as

$$LL_A = \sum_{i=1}^M \log \left( 1 - \Phi_{0,\sigma} [u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)] [u(L_i, \theta) - u(O_i, \theta)] \right) \quad (6)$$

As usual, parameters  $\theta$  and  $\sigma$  are estimated to maximize total log-likelihood  $LL_R + LL_A$ .

#### 2.4. Fechner model of heteroscedastic and truncated random errors

Blavatskyy (2007) extend a Fechner model of heteroscedastic errors by truncating the distribution of random errors so that an individual does not commit transparent errors. An example of such a transparent error is the situation when an individual values a risky lottery more than its highest possible outcome for certain or when an individual values a risky lottery less than its lowest possible outcome for certain (known as a violation of the internality axiom).

In a binary choice between a risky lottery and a monetary amount for certain, a rational individual would always reject the amount if it is smaller than the lowest possible outcome of a risky lottery. Similarly, the individual would always accept the amount if it exceeds the highest possible outcomes of a risky lottery. However, according to a Fechner model presented in Sections 2.2 and 2.3 there is a strictly positive probability that a decision maker commits such a transparent error.

To disregard such transparent errors, the distribution of heteroscedastic Fechner errors is truncated from above and from below. Specifically, the log-likelihood of observing  $N$  decisions when individuals choose a risky lottery  $L_i$  over a monetary amount  $O_i \in [x_i, \bar{x}_i]$  for certain, for all  $i \in \{1, \dots, N\}$ , can be written as

$$LL_R = \sum_{i=1}^N \log \left( \frac{\Phi_{0,\sigma} [u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)] [u(\bar{x}_i, \theta) - u(L_i, \theta)] - \Phi_{0,\sigma} [u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)] [u(O_i, \theta) - u(L_i, \theta)]}{\Phi_{0,\sigma} [u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)] [u(\bar{x}_i, \theta) - u(L_i, \theta)] - \Phi_{0,\sigma} [u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)] [u(\underline{x}_i, \theta) - u(L_i, \theta)]} \right) \quad (7)$$

and the log-likelihood of observing  $M$  decisions when individuals choose a monetary amount  $O_i \in [x_i, \bar{x}_i]$  for certain over a risky lottery  $L_i$ , for all  $i \in \{1, \dots, M\}$ , can be written as

$$LL_A = \sum_{i=1}^M \log \left( \frac{\Phi_{0,\sigma}[u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)][u(O_i, \theta) - u(L_i, \theta)] - \Phi_{0,\sigma}[u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)][u(\underline{x}_i, \theta) - u(L_i, \theta)]}{\Phi_{0,\sigma}[u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)][u(\bar{x}_i, \theta) - u(L_i, \theta)] - \Phi_{0,\sigma}[u(\bar{x}_i, \theta) - u(\underline{x}_i, \theta)][u(\underline{x}_i, \theta) - u(L_i, \theta)]} \right) \quad (8)$$

Note that when a sure monetary amount is exactly equal to the lowest (highest) possible outcome of a risky lottery, an individual chooses the risky lottery with probability one (zero) and she chooses the sure amount with probability zero (one). As usual, parameters  $\theta$  and  $\sigma$  are estimated to maximize combined log-likelihood  $LL_R + LL_A$ .

### 2.5. Random utility model

In a random utility model individual preferences over lotteries are stochastic. They are represented by a probability measure over preference functionals (e.g. Loomes and Sugden, 1995; Regenwetter and Marley, 2001). If preferences are captured by a decision theory with a vector of parameters  $\theta$ , we will assume that one of the parameters  $\theta_R \in \theta$  is normally distributed with mean  $\mu$  and standard deviation  $\sigma$  and the remaining parameters  $\theta_{-R}$  are non-stochastic.<sup>1</sup> Let  $\bar{\theta}_R(\theta_{-R})$  denote the value of parameter  $\theta_R$  such, that given other parameters  $\theta_{-R}$ , an individual is exactly indifferent between a monetary amount  $O$  for certain and a risky lottery  $L$ , i.e.,  $u(L, [\bar{\theta}_R(\theta_{-R}), \theta_{-R}]) = u(O, [\bar{\theta}_R(\theta_{-R}), \theta_{-R}])$ . Without a loss of generality, we can assume that for all parameter values above this threshold, i.e.,  $\theta_R > \bar{\theta}_R(\theta_{-R})$ , an individual prefers the sure amount over the risky lottery (otherwise we can always define a new parameter  $\theta_R^* = -\theta_R$ ).

The log-likelihood of observing  $N$  decisions when individuals choose a risky lottery  $L_i$ ,  $i \in \{1, \dots, N\}$ , over a monetary amount  $O_i$  for certain, can then be written as

$$LL_R = \sum_{i=1}^N \log(\Phi_{\mu,\sigma}[\bar{\theta}_R(\theta_{-R})]) \quad (9)$$

and the log-likelihood of observing  $M$  decisions when individuals choose a monetary amount  $O_i$ ,  $i \in \{1, \dots, M\}$ , for certain over a risky lottery  $L_i$ , can be written as

$$LL_A = \sum_{i=1}^M \log(1 - \Phi_{\mu,\sigma}[\bar{\theta}_R(\theta_{-R})]) \quad (10)$$

Note that the probability that an individual chooses a risky lottery over a monetary amount is simply the probability of observing preferences characterized by parameter  $\theta_R < \bar{\theta}_R(\theta_{-R})$ . Similarly, the likelihood that an individual chooses the sure amount is just the likelihood of observing parameter  $\theta_R > \bar{\theta}_R(\theta_{-R})$ . Parameters  $\theta_{-R}$ ,  $\mu$  and  $\sigma$  are estimated to maximize combined log-likelihood  $LL_R + LL_A$ . Note that our estimation of expected utility theory with a one-parameter

<sup>1</sup> We do not assume that all parameters in vector  $\theta$  are jointly normally distributed because maximum likelihood estimation of such a (nonlinear) model requires numerical integration, which significantly impacts the speed of computations. Even with only one stochastic parameter, the estimation of rank-dependent utility and disappointment aversion theory embedded into a random utility model took as long as 2 months on Pentium 4 3.0 GHz CPU.

utility function embedded in a random utility model is equivalent to standard models of unobserved heterogeneity (when a researcher assumes that the index of risk aversion is normally distributed across the population from which contestants were drawn and the mean and variance of this distribution are estimated).

## 2.6. Luce choice model (strict utility model)

Luce (1959) has proposed a stochastic choice model (known as strict utility model) where the probability that a risky lottery  $L$  is chosen over a monetary amount  $O$  for certain can be written as  $u(L, \theta)^{1/\mu} / [\mu(L, \theta)^{1/\mu} + u(O, \theta)^{1/\mu}]$ , where  $\mu > 0$  is a noise parameter. This model has been recently popularized by Holt and Laury (2002). It is well known (e.g., Theorem 30 in Luce and Suppes, 1965) that the Luce choice model can be rewritten as a Fechner model of homoskedastic random errors so that the probability that a risky lottery  $L$  is chosen over amount  $O$  for certain is given by  $\Lambda_\mu[\tilde{u}(L, \theta) - \tilde{u}(O, \theta)]$ , where  $\Lambda_\mu[x] = 1/(1 + \exp(-x/\mu))$  is the cdf of the logistic distribution and  $\tilde{u}(\cdot) = \log u(\cdot)$ .

Note that in this model utility scale is determined up to a multiplication by a positive constant (Luce and Suppes, 1965, p. 335). All decision theories that we consider in this paper employ utility functions that are unique up to affine transformation; i.e., if utility function  $u(\cdot)$  represents individual preferences then utility function  $u'(\cdot) = au(\cdot) + b$  represents the same preferences for any  $a > 0$  and any  $b$ . A decision theory with utility function which is determined up to an affine transformation cannot be estimated in conjunction with the Luce choice model. If we estimate such decision theory with utility function  $u(\cdot)$  embedded in the Luce choice model, we receive different results compared to the estimation of the same decision theory with utility function  $u'(\cdot) = au(\cdot) + b$ ,  $a > 0$ ,  $b \neq 0$ . In other words, arbitrary normalization of utility function (in particular, a shift in the utility scale by a constant  $b \neq 0$ ) affects estimated parameters of a decision theory. Therefore, we do not estimate decision theories embedded in the Luce choice model.

## 3. DATA

We use data from a natural experiment provided by the television show *Deal or No Deal*. In this show, monetary prizes ranging from 1 cent to half-a-million euros are randomly allocated across identical boxes. The list of potential prizes is common knowledge but their allocation across boxes is kept secret. A contestant is endowed with one box and she has to open the remaining boxes one by one. Once a box is opened, the prize sealed inside is publicly revealed and deleted from the list of possible prizes.

The more boxes are opened, the more information the contestant receives about the distribution of possible prizes inside her box. After opening several boxes the contestant receives an offer from the 'bank' (the timing of 'bank' offers is presented in Figures 1 and 2). The 'bank' offers either a sure monetary amount in exchange for the contestant's box or the possibility to swap the contestant's box for any of the remaining unopened boxes. If the contestant rejects the sure amount, and regardless of the contestant's decision on the swap offer, she has to continue opening boxes one by one until the 'bank' makes another offer or all boxes are opened.

Monetary offers are fairly predictable across episodes and follow a general pattern. When many boxes remain unopened, the 'bank' offers monetary amounts, which are significantly below the

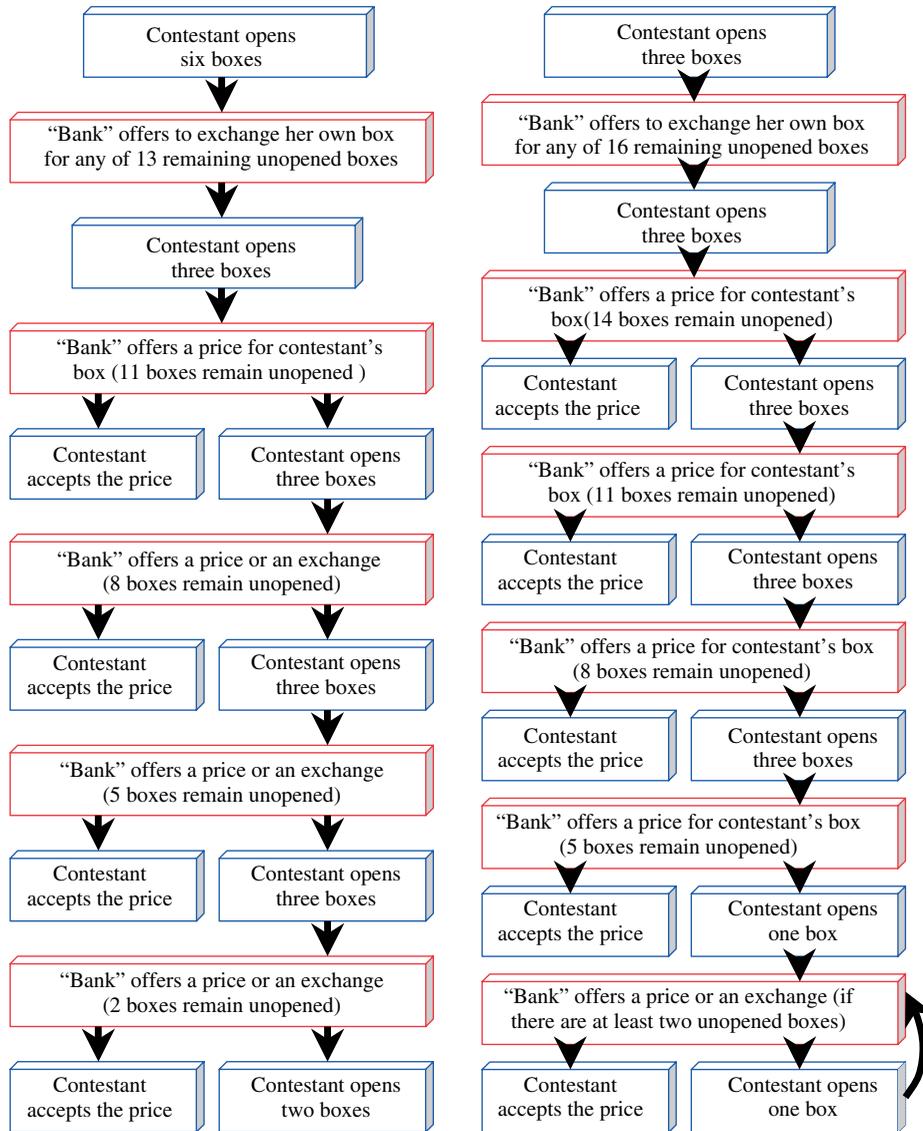


Figure 1. Timing of 'bank' offers in *Affari Tuoi* television episodes before 9 February 2006 (left chart) and starting from 9 February 2006 (right chart). This figure is available in color online at [wileyonlinelibrary.com/journal/jae](http://wileyonlinelibrary.com/journal/jae)

expected value of possible prizes. As more and more boxes are opened, the gap between the expected value of remaining prizes and an offer decreases. The game terminates either when the contestant accepts a sure monetary amount or when all boxes are opened. In the latter case, the contestant leaves with the content of the box which is opened last.

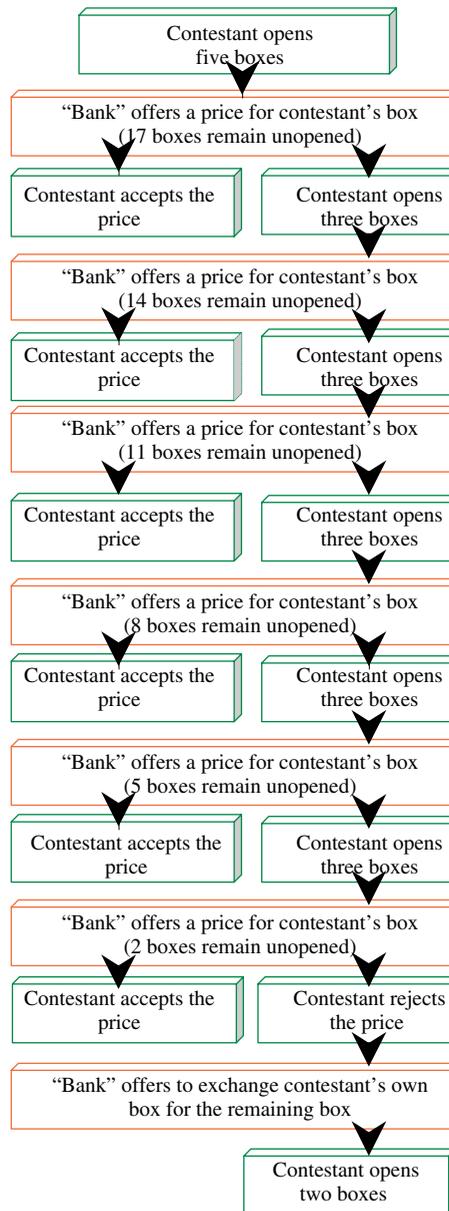


Figure 2. Timing of 'bank' offers in *Deal or No Deal UK* television episodes. This figure is available in color online at [wileyonlinelibrary.com/journal/jae](http://wileyonlinelibrary.com/journal/jae)

Our dataset consists of 114 episodes of the Italian version of the show (*Affari Tuoi*) broadcast on the first channel of the Italian television RAI Uno from 20 September 2005 to 4 March 2006 and 518 episodes of the British version of the show (*Deal or No Deal UK*) aired on the Channel

4 of the British television from 31 October 2005 to 20 August 2007.<sup>2</sup> In every episode, only one contestant plays the game and she decides on at least one monetary offer.<sup>3</sup> Variables that we use in the empirical estimation are the contestant's decision to accept or to reject an offer, the 'bank' offer itself, and the list of possible prizes that the contestant can potentially win if she rejects the 'bank' offer.

45.6% of Italian contestants and 50.2% of British contestants are male. Average age is 46.3 years for Italian contestants and 41 years for British contestants. 14.0% and 78.9% of Italian contestants and 48.3% and 50.8% of British contestants are correspondingly single and married. Representatives of all administrative regions of Italy and representatives of 22 administrative regions of the UK appear in our recorded dataset. Thus this natural experiment employs a more diverse subject pool than conventional pools, composed primarily of undergraduate students.

*Ex ante* expected value of monetary prizes that are allocated across boxes is €52,295 in *Affari Tuoi* and £25,712 in *Deal or No Deal UK*. Average and median earnings of Italian contestants are €29,516 and €19,000 respectively. Average and median earnings of British contestants are £16,069 and £13,000 correspondingly. Overall, obtaining a similar dataset under conventional laboratory conditions would be a highly ambitious project, since it would require a total budget of almost 10 million euros.

The natural experiment allows us to use a more diverse subject pool and significantly higher incentives than in a typical laboratory experiment. However, it has a disadvantage that we do not have any control over sure amounts that are offered to the contestants. Moreover, a precise mechanism of setting 'bank' offers is not revealed to the public in official show regulations. However, a simple regression analysis shows that the variability in 'bank' monetary offers is largely explained by only two variables: the expected value of possible prizes and the number of unopened boxes left in the game.

Tables I and II show the results of the ordinary least squares regression of a natural logarithm of 'bank' monetary offers on the natural logarithm of the expected value of possible prizes.<sup>4</sup> We conduct a separate regression for different rounds of the game (characterized by the number of unopened boxes left in the game). Tables I and II show that the expected value of possible prizes explains a significant portion of the variability in 'bank' offers (particularly when few boxes remain unopened). Regression coefficients differ significantly across the columns of Tables I and II, i.e., the number of unopened boxes is also a significant explanatory variable for 'bank' offers.

We also regress 'bank' offers on other lottery-specific variables (median prize, the standard deviation of prizes and the prize hidden inside the contestant's box) and socio-demographic characteristics of contestants (gender, age, marital status and region). Among these variables, only regression coefficient on the standard deviation of possible prizes is statistically significant (the

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<sup>2</sup> Blavatsky and Pogrebna (2008) and Pogrebna (2008) describe television shows *Affari Tuoi* and *Deal or No Deal UK* in detail. Blavatsky and Pogrebna (2008) also provide a detailed review of numerous working papers that analyzed data from different national versions of the television show *Deal or No Deal*.

<sup>3</sup> In our recorded sample only one *Affari Tuoi* contestant accepted the first monetary offer from the 'bank' (€18,000). Ten contestants accepted the second monetary offer that they received from the 'bank'. Thirty-four contestants accepted their third monetary offer. All remaining contestants received from four to seven monetary offers. In *Deal or No Deal UK* all contestants rejected the first two monetary offers, nine contestants accepted the third monetary offer and all remaining contestants received from four to seven monetary offers.

<sup>4</sup> Note that in Tables I and II we have sometimes fewer observations for earlier rounds of the game (when there are more unopened boxes). This happens because the 'bank' occasionally makes non-monetary offers and gives a contestant the opportunity to swap her box for any of the remaining unopened boxes. If the 'bank' makes a non-monetary offer in earlier rounds of the game, we have a missing observation.

Table I. Ordinary least squares regression  $\ln \mathbf{O} = \beta_0 + \beta_1 \ln \mathbf{EV} + \varepsilon$  of 'bank' monetary offers  $\mathbf{O}$  on expected value  $\mathbf{EV}$  of possible prizes in *Affari Tuoi*

Number of unopened boxes	2	3	4	5	8	11
Constant $\hat{\beta}_0$	-0.9796 (0.4355)	-3.0064 (1.5389)	0.6209 (0.3581)	0.6610 (0.3282)	-0.5971 (0.5960)	-5.2172 (0.9828)
Expected value $\hat{\beta}_1$	1.0572 (0.0470)	1.1960 (0.1524)	0.8479 (0.0337)	0.8395 (0.0309)	0.9308 (0.0554)	1.3314 (0.0912)
$R^2$	0.9285	0.8725	0.9829	0.8870	0.7216	0.6555
Observations	41	11	13	96	111	114

Table II. Ordinary least squares regression  $\ln \mathbf{O} = \beta_0 + \beta_1 \ln \mathbf{EV} + \varepsilon$  of 'bank' monetary offers  $\mathbf{O}$  on expected value  $\mathbf{EV}$  of possible prizes in *Deal or No Deal UK*

Number of unopened boxes	2	5	8	11	14	17
Constant $\hat{\beta}_0$	-0.3675 (0.0533)	-0.9086 (0.1667)	-1.0210 (0.2569)	-1.7693 (0.4953)	-3.5305 (0.8990)	-14.3036 (2.2340)
Expected value $\hat{\beta}_1$	1.0003 (0.0065)	1.0317 (0.0173)	1.0255 (0.0259)	1.0826 (0.0495)	1.2277 (0.0893)	2.2340 (0.1458)
$R^2$	0.9915	0.9034	0.7614	0.4815	0.2684	0.3208
Observations	207	381	492	517	517	499

more dispersed are the prizes, the lower is the offer). Notably, regression coefficient on the prize inside the contestant's box is never statistically significant, i.e. there is no information content in 'bank' offers. Details of this regression analysis are given in Table III in Blavatskyy and Pogrebna (2008, p. 411).

#### 4. ESTIMATION PROCEDURE

In the *Deal or No Deal* television show contestants face a series of binary choices between a degenerate lottery ('bank' monetary offer for certain) and a risky lottery. We will consider *Deal or No Deal* as a dynamic decision problem. In this problem, contestants evaluate a risky lottery taking into account the expectation of future 'bank' offers that they will receive if they reject the current offer.

In the last round of the game, a contestant facing prizes  $x_1$  and  $x_2$  hidden in two unopened boxes perceives them as a risky lottery  $L(x_1, 1/2; x_2, 1/2)$ . In all other rounds of the game, a contestant facing prizes  $\mathbf{x} = \{x_1, \dots, x_n\}$  hidden in  $n > 2$  unopened boxes perceives them as a risky lottery  $L(\mathbf{x})$ . Let  $m$  denote the number of boxes that the contestant has to open before the next 'bank' offer is made ( $m$  is either 1 or 3 in *Affari Tuoi* and  $m = 3$  in *Deal or No Deal UK*). There are  $C_{n-m}^n = n!/(m!(n-m)!)$  combinations of prizes  $\mathbf{x}$  that the contestant can face when the next offer is made. Let us denote these combinations by  $\mathbf{x}_1, \dots, \mathbf{x}_{C_{n-m}^n}^n$ . Lottery  $L(\mathbf{x})$  is then recursively

Table III. Maximum likelihood estimates of parameters of decision theories embedded in different models of stochastic choice and obtained log-likelihood values (data from *Affari Tuoi*)

Decision theory	Model of stochastic choice				
	Tremble	Fechner model of random errors			Random utility
		Homoscedastic	Heteroscedastic	Heteroscedastic and truncated	
RN	$p = 0.1603$ $LL = -179.2$	$\sigma = 68, 685$ $LL = -225.8$	$\sigma = 0.1697$ $LL = -196.8$	$\sigma = 0.3267$ $LL = -174.2$	$LL = -\infty$
EUT + CRRA	$p = 0.1603$ $r = 0$ $LL = -179.2$	$\sigma = 176.74$ $r = 0.4542$ $LL = -205.5$	$\sigma = 0.1413$ $r = 0.1929$ $LL = -193.1$	$\sigma = 1.6139$ $r = 0.2820$ $LL = -165.8$	$\mu = -0.1201$ $\sigma = 0.8935$ $LL = -166.04$
EUT + EP	$p = 0.1063$ $r = -0.3389$ $\alpha = -0.0013$ $LL = -168.9$	$\sigma = 52, 612$ $r = -0.2213$ $\alpha = 2.42 \times 10^{-6}$ $LL = -174.7$	$\sigma = 0.1487$ $r = -0.2052$ $\alpha = 2.37 \times 10^{-6}$ $LL = -171.8$	$\sigma = 1.7651$ $r = -0.1564$ $\alpha = 2.2 \times 10^{-6}$ $LL = -157.4$	$\mu = -0.7846$ $\sigma = 1.1035$ $\alpha = 1.0 \times 10^{-6}$ $LL = -153.8$
RT (SSB)	$p = 0.1603$ $r = 0$ $\delta = 1$ $LL = -179.2$	$\sigma = 24.828$ $r = -0.0270$ $\delta = 0.4127$ $LL = -197.7$	$\sigma = 0.1401$ $r = 0.2107$ $\delta = 1.0548$ $LL = -193.1$	$\sigma = 4.4475$ $r = 0.0371$ $\delta = 0.6201$ $LL = -164.2$	$\mu = -0.2926$ $\sigma = 1.0905$ $\delta = 0.7420$ $LL = -165.94$
YDM	$p = 0.1603$ $\gamma = 1$ $LL = -179.2$	$\sigma = 38, 430$ $\gamma = 0.3113$ $LL = -203.1$	$\sigma = 0.1743$ $\gamma = 0.5347$ $LL = -177.2$	$\sigma = 0.3174$ $\gamma = 1.8288$ $LL = -168.3$	$\mu = 1.3618$ $\sigma = 1.0818$ $LL = -173.4$
RDU	$p = 0.1603$ $r = 0$ $\gamma = 1$ $LL = -179.2$	$\sigma = 273.15$ $r = 0.4244$ $\gamma = 0.6192$ $LL = -194.0$	$\sigma = 0.1640$ $r = 0.0204$ $\gamma = 0.5133$ $LL = -176.9$	$\sigma = 1.1057$ $r = 0.2712$ $\gamma = 1.1106$ $LL = -165.8$	$\mu = -0.1336$ $\sigma = 0.9089$ $\gamma = 1.0201$ $LL = -166.03$
DAT	$p = 0.1603$ $r = 0$ $\beta = 0$ $LL = -179.2$	$\sigma = 3.3624$ $r = 0.9619$ $\beta = -0.9857$ $LL = -171.9$	$\sigma = 0.6134$ $r = 0.6286$ $\beta = -0.9999$ $LL = -169.6$	$\sigma = 9.6590$ $r = 0.2902$ $\beta = -0.0213$ $LL = -165.8$	$\mu = -0.0903$ $\sigma = 0.8754$ $\beta = -0.0230$ $LL = -165.99$

defined by

$$\begin{aligned}
 L(\mathbf{x}) = & \frac{1 - \hat{\pi}_{n-m}}{C_{n-m}^n} \sum_{i=1}^{C_{n-m}^n} [L(\mathbf{x}_i)I(u(L(\mathbf{x}_i), \theta) \geq u(\hat{O}(\mathbf{x}_i), \theta)) + \hat{O}(\mathbf{x}_i)I(u(L(\mathbf{x}_i), \theta) < u(\hat{O}(\mathbf{x}_i), \theta))] \\
 & + \frac{\hat{\pi}_{n-m}}{C_{n-m}^n} \sum_{i=1}^{C_{n-m}^n} L(\mathbf{x}_i)
 \end{aligned}
 \tag{11}$$

where  $\hat{O}(\mathbf{x}_i)$  is an expectation of a future ‘bank’ offer for  $\mathbf{x}_i$  prizes left in unopened boxes and  $\hat{\pi}_{n-m}$  is the expected probability that the ‘bank’ offers a swap option instead of a monetary amount at the stage when  $n - m$  boxes remain unopened.<sup>5</sup> Note that we use a fully rational forward-looking algorithm for calculating the reduced form of lottery  $L(\mathbf{x})$ .

<sup>5</sup> For any two lotteries  $L_1(y_1, p_1; \dots; y_k, p_k)$  and  $L_2(z_1, q_1; \dots; z_l, q_l)$  a compound lottery  $\alpha L_1 + (1 - \alpha)L_2$ ,  $\alpha \in [0, 1]$ , is defined in the usual way—it yields outcome  $y_i$  with probability  $\alpha \cdot p_i$ ,  $i \in \{1, \dots, k\}$ , and outcome  $z_j$  with probability  $(1 - \alpha) \cdot q_j$ ,  $j \in \{1, \dots, l\}$ .

If a decision theory satisfies the independence axiom, e.g., expected utility theory, the utility of lottery  $L(\mathbf{x})$  can be conveniently calculated through the Bellman optimality equation:

$$u(L(\mathbf{x}), \theta) = \frac{1}{C_{n-m}^n} \sum_{i=1}^{C_{n-m}^n} [(1 - \hat{\pi}_{n-m}) \max\{u(L(\mathbf{x}_i), \theta), u(\hat{O}(\mathbf{x}_i), \theta)\} + \hat{\pi}_{n-m} u(L(\mathbf{x}_i), \theta)] \quad (12)$$

However, this equation does not hold for generalized non-expected utility theories.

Recall that the ‘bank’ offers depend only on the expected value of the remaining prizes and the number of the remaining prizes. Therefore, the expectation of future ‘bank’ offers  $\hat{O}(\mathbf{x}_i)$  is calculated by means of a simple regression  $\ln \hat{O}(\mathbf{x}_i) = \hat{\beta}_0 + \hat{\beta}_1 \ln EV(\mathbf{x}_i)$ . In this regression the estimates of the coefficients  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are taken from the column in Tables I and II corresponding to  $n - m$  number of unopened boxes and  $EV(\mathbf{x}_i)$  is the expected value of prizes  $\mathbf{x}_i$ ,  $i \in \{1, \dots, C_{n-m}^n\}$ . Probability  $\hat{\pi}_{n-m}$  is equal to the actual probability with which the ‘bank’ made a swap offer in our recorded sample. In *Deal or No Deal UK* the ‘bank’ made swap offers instead of a monetary offer only in the first offer in the game. Therefore, probability  $\hat{\pi}_{n-m}$  is set equal to zero for the British dataset.

Estimation of a random utility model in a dynamic decision problem deserves special attention. In a random utility model, preferences of a decision maker change each time a new decision is made. Thus, it is not immediately clear how such an individual forms expectations about own future decisions. We assume the simplest possible scenario that contestants expect to keep on maximizing their current preferences (as captured by current parameters of decision theory) while considering decision making in the future.

Since every contestant makes only up to seven decisions, we adopt a representative agent approach and estimate every pair of decision theory and stochastic choice model on the aggregate data. The utility of lotteries is evaluated through the parametric form of seven decision theories: risk neutrality (RN), expected utility theory (EUT) with constant relative risk aversion (CRRA) and expo-power (EP) utility function, regret theory (RT) or skew-symmetric bilinear utility theory (SSB), Yaari’s dual model (YDM), rank-dependent utility theory (RDU) and disappointment aversion theory (DAT). Parametric functional forms of these theories are described in the Appendix.

The free parameters of decision theories are estimated by maximizing the log-likelihood function of a selected model of stochastic choice described in Section 2. Nonlinear optimization was implemented in the MATLAB 7.2 package (based on the Nelder–Mead simplex algorithm).

## 5. RESULTS

### 5.1. Results for *Affari Tuoi*

Table III shows maximum likelihood estimates of the parameters of decision theories embedded in five different models of stochastic choice and a corresponding log-likelihood of observing actual decisions of *Affari Tuoi* contestants. A comparison of parameter estimates of the same decision theory across different columns of Table III demonstrates our first somewhat unexpected result:

- *Result 1.* Estimated parameters of the same decision theory differ substantially, depending on which model of stochastic choice the theory is embedded in.

For example, estimates of EUT with CRRA utility function indicate risk neutrality in a tremble model, risk aversion in a Fechner model, and risk seeking (on average) in a random utility model. Estimates of YDM show no probability distortions in a tremble model, overweighting of small probabilities and underweighting of large probabilities in a Fechner model with homoskedastic and heteroscedastic errors, and underweighting of small probabilities and overweighting of large probabilities in the remaining models.

Estimates of EUT with EP utility function show that  $r < 0$  and  $\alpha > 0$  for all models except for a tremble model; i.e., contestants exhibit increasing relative risk aversion and increasing absolute risk aversion. Estimates of RT (SSB) are consistent with the assumption of regret aversion only in a Fechner model with heteroscedastic errors. Under RDU, the utility function is estimated to be linear in a tremble model, concave in a Fechner model, and convex (on average) in a random utility model.

The estimated coefficient of the probability weighting function in YDM and RDU embedded in a Fechner model with heteroscedastic truncated errors is notably higher than one, which indicates that the probability weighting function is S-shaped and convex nearly on its entire domain. We also observe this unexpected result in the British dataset (see Table V below) and it is worthwhile to explain its cause. In a Fechner model with heteroscedastic truncated errors, random errors are likely to increase utility of lotteries that deliver improbable gains and the same random errors are likely to decrease utility of lotteries that deliver probable gains (e.g., Blavatsky, 2007). Traditional inverse S-shaped probability weighting function of RDU fulfils exactly the same role.

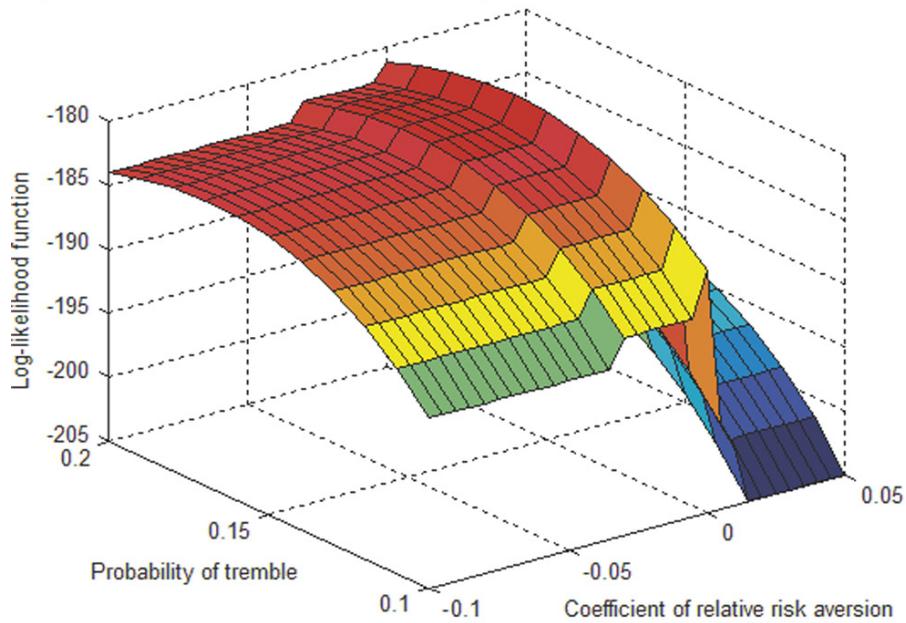
Thus, RDU embedded in a Fechner model with heteroscedastic truncated errors has two built-in components that both can lead to systematic overweighting of small probabilities and underweighting of large probabilities. In a maximum likelihood estimation of such a model, there is no need to keep both components at work. If the estimated standard deviation of random errors converges to zero, the systematic effect of random errors disappears. If the estimated coefficient of the probability weighting function gets larger than one, the traditional effect of probability distortions disappears. Table III (and Table V below) show that the second possibility yields a better fit to the data.

Table III also shows that for all theories that have RN as a special case, the estimates are the same as for RN when they are embedded in a tremble model (note that EUT with EP utility function does not have RN as a special case). This accidental result is driven by a mass point in 'bank' offers. In our recorded sample of *Affari Tuoi* nine offers (2.2%) are exactly equal to the expected value of the prizes (four of these offers were accepted). RN embedded in a tremble model yields a log-likelihood of  $9 \log(1/2)$ . Any other decision theory that does not assume perfect risk neutrality yields a log-likelihood of  $4 \log(p) + 5 \log(1 - p)$  or  $5 \log(p) + 4 \log(1 - p)$ , where  $p$  is a tremble probability. For a small  $p$ , each of these log-likelihood values is significantly lower than  $9 \log(1/2)$ , which handicaps non-RN theories. For example, Figure 3 shows that the log-likelihood functions of EUT and YDM have a sizeable spike corresponding to a special case of RN.

To compare the fit of various combinations of a decision theory with a model of stochastic choice, we use a likelihood ratio test for nested models and Vuong likelihood ratio test for non-nested models or overlapping models (e.g., Vuong, 1989). Loomes *et al.* (2002, p. 128) describe the application of the Vuong likelihood ratio test to the selection between different stochastic choice models.

Table IV compares the fit of two most successful decision theories within every model of stochastic choice (for each model we selected two theories that outperformed other theories

Log-likelihood Function of EUT with CRRA Utility Function Embedded into a Tremble Model



Log-likelihood Function of Yaari's Dual Model Embedded into a Tremble Model

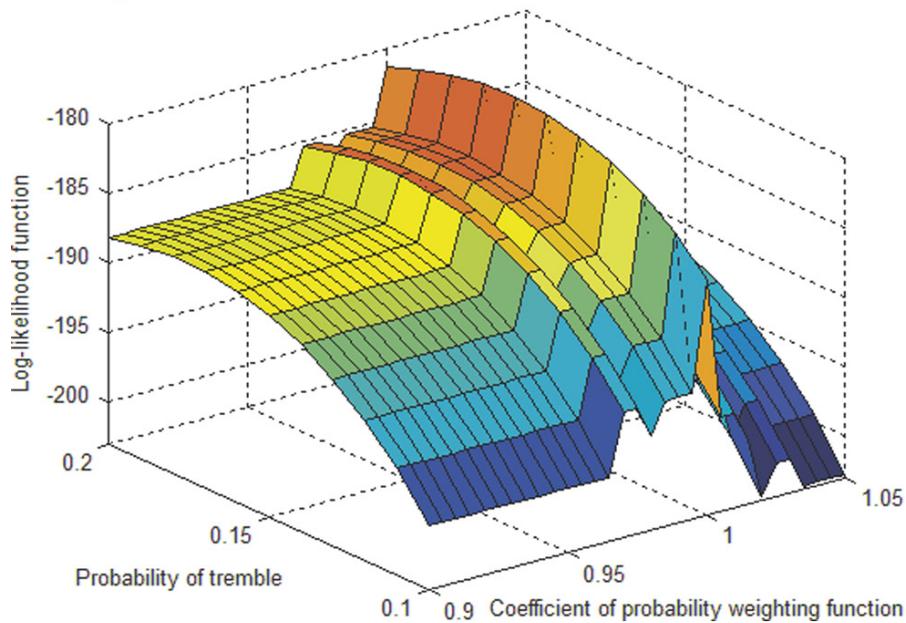


Figure 3. Log-likelihood functions of EUT and YDM embedded into a tremble model. This figure is available in color online at [wileyonlinelibrary.com/journal/jae](http://wileyonlinelibrary.com/journal/jae)

Table IV. Vuong likelihood ratio test ( $p$ -value) of selected pairs of a decision theory and a stochastic choice model (data from *Affari Thoi*). A significantly positive (negative) value indicates that a row (column) pair is closer to the true data-generating process than a column (row) pair. Akaike Information Criterion is used to adjust for a smaller number of parameters in RN

Stochastic choice model	→	Tremble model		Fechner model, homoscedastic errors		Fechner model, heteroscedastic errors		Fechner model, heteroscedastic truncated errors		Random utility model	
		EUT+EP	RN	DAT	EUT+EP	DAT	EUT+EP	DAT	EUT+EP	DAT	EUT+EP
Tremble model	EUT+EP	—	0.9552 (0.1698)	0.3513 (0.3627)	6.7160 (0.0000)	0.0869 (0.4654)	0.3159 (0.3760)	-1.4208 (0.0777)	-0.5242 (0.3001)	-0.6348 (0.2628)	-0.2862 (0.3874)
	RN	-0.9552 (0.1698)	—	-1.0948 (0.1368)	5.1655 (0.0000)	-1.9433 (0.0260)	-0.5816 (0.2804)	-2.4603 (0.0069)	-2.4175 (0.0078)	-1.5190 (0.0644)	-2.0415 (0.0206)
Fechner model, homoscedastic	DAT	-0.3513 (0.3627)	1.0948 (0.1368)	—	5.6689 (0.0000)	-0.8866 (0.1877)	-0.0176 (0.4930)	-2.6116 (0.0045)	-5.5139 (0.0000)	-1.0573 (0.1452)	-1.1736 (0.1203)
	EUT+EP	-6.7160 (0.0000)	-5.1655 (0.0000)	-5.6689 (0.0000)	—	-5.7487 (0.0000)	-7.9990 (0.0000)	-7.1385 (0.0000)	-6.2497 (0.0000)	-6.0319 (0.0000)	-5.9889 (0.0000)
Fechner model, heteroscedastic	DAT	-0.0869 (0.4654)	1.9433 (0.0260)	0.8866 (0.1877)	5.7487 (0.0000)	—	0.2528 (0.4002)	-1.8456 (0.0325)	-1.6047 (0.0543)	-0.7727 (0.2199)	-0.7919 (0.2142)
	EUT+EP	-0.3159 (0.3760)	0.5816 (0.2804)	0.0176 (0.4930)	7.9990 (0.0000)	-0.2528 (0.4002)	—	-1.9501 (0.0256)	-0.9556 (0.1696)	-0.8438 (0.1994)	-0.7006 (0.2418)
Fechner, heteroscedastic truncated	EUT+EP	1.4208 (0.0777)	2.4603 (0.0069)	2.6116 (0.0045)	7.1385 (0.0000)	1.8456 (0.0325)	1.9501 (0.0256)	—	1.3797 (0.0838)	-0.5785 (0.2815)	1.0409 (0.1490)
	RT (SSB)	0.5242 (0.3901)	2.4175 (0.0078)	5.5139 (0.0000)	6.2497 (0.0000)	1.6047 (0.0543)	0.9556 (0.1696)	-1.3797 (0.0838)	—	-0.1764 (0.4300)	0.3388 (0.3674)
Random utility model	EUT+EP	0.6348 (0.2628)	1.5190 (0.0644)	1.0573 (0.1452)	6.0319 (0.0000)	0.7727 (0.2199)	0.8438 (0.1994)	0.5785 (0.2815)	0.1764 (0.4300)	—	0.3396 (0.3671)
	RT (SSB)	0.2862 (0.3874)	2.0415 (0.0206)	1.1736 (0.1203)	5.9889 (0.0000)	0.7919 (0.2142)	0.7006 (0.2418)	-1.0409 (0.1490)	-0.3388 (0.3674)	-0.3396 (0.3671)	—

embedded in the *same* stochastic choice model). Our second important result can be summarized as follows:

- *Result 2.* For every decision theory, the best fit to the data is obtained when this theory is embedded into a Fechner model with heteroscedastic truncated errors or a random utility model.

Perhaps not surprisingly, for every model of stochastic choice but a tremble model, the worst fit to the data is obtained when this model is combined with RN. However, there is no single decision theory that provides the best goodness of fit in all models.

- *Result 3.* Decision theory with the best goodness of fit depends on the selected model of stochastic choice. For example, it is EUT with EP utility function in a tremble model, and DAT in a Fechner model of homoskedastic errors.

Result 3 implies that any comparison of decision theories should be reported with a reference to the model of stochastic choice that is being used. Otherwise, a hunter for a descriptive decision theory would claim that DAT performs better than other theories in a Fechner model with homo- or heteroscedastic errors. At the same time, a researcher who employs a tremble model, a Fechner model with heteroscedastic truncated errors or a random utility model would claim that EUT with EP utility function performs best.

## 5.2. Results for *Deal or No Deal UK*

Table V shows maximum likelihood estimates for a dynamic decision problem in *Deal or No Deal UK*. Result 1 clearly holds in Table V. Maximum likelihood estimate of EUT with CRRA utility function shows that *Deal or No Deal UK* contestants are nearly risk-neutral in a tremble model, risk averse (with a coefficient of relative risk aversion being between 0.13 and 0.23) in a Fechner model and risk seeking (on average) in a random utility model. Estimate of EUT with EP utility function shows that contestants exhibit increasing relative risk aversion ( $\alpha > 0$ ) in all models except for a tremble model. They also reveal increasing absolute risk aversion ( $r < 0$ ) in all models.

Estimation of RDU shows that contestants have an inverse S-shaped probability weighting function (with coefficient  $\gamma$  between 0.52 and 0.78) and a concave utility function (with a coefficient of relative risk aversion between 0.06 and 0.24) in all models except for a Fechner model with heteroscedastic truncated errors. Estimation of DAT shows that contestants are disappointment seeking in a Fechner model with homoskedastic or heteroscedastic errors and disappointment averse in other models.

As in the Italian dataset, Result 2 holds: every decision theory describes the decisions of *Deal or No Deal UK* contestants most accurately when it is embedded in a Fechner model with heteroscedastic truncated errors. For every model of stochastic choice, the worst fit to the data is obtained when this model is combined with RN.

Table VI compares the goodness of fit of selected pairs of a decision theory and a stochastic choice model (for each model we selected two theories that outperformed other theories embedded in the *same* stochastic choice model). Result 3 clearly holds in Table VI. In a tremble model and a Fechner model with heteroscedastic truncated errors, EUT with EP utility function and RT

Table V. Maximum likelihood estimates of parameters of decision theories embedded in different models of stochastic choice and obtained log-likelihood values (data from *Deal or No Deal UK*)

Decision theory	Model of stochastic choice				
	Tremble	Fechner model of random errors			Random utility
		Homoscedastic	Heteroscedastic	Heteroscedastic and truncated	
RN	$p = 0.1470$ LL = -1093	$\sigma = 17,718$ LL = -1265	$\sigma = 0.1078$ LL = -1176	$\sigma = 0.4110$ LL = -890.8	LL = $-\infty$
EUT + CRRA	$p = 0.1450$ $r = 0.0520$ LL = -1082	$\sigma = 841.46$ $r = 0.2368$ LL = -1170	$\sigma = 0.0827$ $r = 0.1707$ LL = -1129	$\sigma = 1.0295$ $r = 0.1328$ LL = -886.5	$\mu = -0.1729$ $\sigma = 0.8160$ LL = -923.8
EUT + EP	$p = 0.0940$ $r = -0.3567$ $\alpha = -1.4 \times 10^{-3}$ LL = -962.7	$\sigma = 14,398,643$ $r = -0.8413$ $\alpha = 1.1 \times 10^{-8}$ LL = -989.9	$\sigma = 0.2036$ $r = -1.0531$ $\alpha = 1.6 \times 10^{-9}$ LL = -983.7	$\sigma = 5.8405$ $r = -0.2408$ $\alpha = 1.3 \times 10^{-6}$ LL = -848.9	$\mu = -2.2498$ $\sigma = 2.4782$ $\alpha = 6.2 \times 10^{-7}$ LL = -1037
RT (SSB)	$p = 0.1424$ $r = 0.3144$ $\delta = 2.3246$ LL = -1069	$\sigma = 2614.3$ $r = 0.3444$ $\delta = 1.2903$ LL = -1155	$\sigma = 0.0730$ $r = 0.3095$ $\delta = 1.5067$ LL = -1102	$\sigma = 0.2180$ $r = -0.6095$ $\delta = 34.3816$ LL = -827.6	$\mu = 0.1524$ $\sigma = 0.4816$ $\delta = 1.8155$ LL = -904.9
YDM	$p = 0.1462$ $\gamma = 0.5217$ LL = -1087.1	$\sigma = 12,461$ $\gamma = 0.4034$ LL = -1056	$\sigma = 0.0977$ $\gamma = 0.4883$ LL = -936.95	$\sigma = 0.3643$ $\gamma = 1.4346$ LL = -869.9	$\mu = 1.7452$ $\sigma = 1.1316$ LL = -942.0
RDU	$p = 0.1432$ $r = 0.1837$ $\gamma = 0.6319$ LL = -1073	$\sigma = 824.75$ $r = 0.2370$ $\gamma = 0.5572$ LL = -1020	$\sigma = 0.0928$ $r = 0.0686$ $\gamma = 0.5223$ LL = -934.7	$\sigma = 0.2321$ $r = -0.3095$ $\gamma = 2.5444$ LL = -865.7	$\mu = 0.0253$ $\sigma = 0.5468$ $\gamma = 0.7747$ LL = -907.4
DAT	$p = 0.1439$ $r = -0.1521$ $\beta = 0.3254$ LL = -1077	$\sigma = 0.6588$ $r = 1.1129$ $\beta = -0.9999$ LL = -884.6	$\sigma = 0.4416$ $r = 0.7263$ $\beta = -0.9999$ LL = -955.1	$\sigma = 0.2454$ $r = -0.2501$ $\beta = 1.7519$ LL = -869.5	$\mu = -0.0582$ $\sigma = 0.4267$ $\beta = 0.2127$ LL = -952.6

(SBB) are most successful in describing the decisions of contestants. In a Fechner model with homoskedastic errors, the decisions of *Deal or No Deal UK* contestants are best predicted by DAT or EUT with EP utility function. In a Fechner model with heteroscedastic errors RDU gives the best fit to the data. Finally, in a random utility model, RT (SSB) and RDU are most successful in describing the decisions of contestants. The best fit to the data across all pairs of decision theories and models of stochastic choice is provided by RT (SBB) or EUT with EP utility embedded in a Fechner model with heteroscedastic truncated errors (at 0.1% significance level).

EUT with CRRA utility function always yields significantly worse fit to the data than EUT with the EP utility function, except when EUT is embedded in a random utility model in the *Deal or No Deal UK* dataset. Thus constant relative risk aversion is ill suited for decision problems that involve both small and large monetary outcomes. The estimated coefficient  $r$  of EUT with the EP utility function is always significantly different from zero. Thus, constant absolute risk aversion also is inappropriate functional form for dynamic decision problems that involve small and large monetary outcomes.

Note that when decision theories are embedded in a Fechner model of homoscedastic errors, the best fit to the data is provided by DAT. This confirms the finding of Camerer and Ho (1994), who re-examine the data from 11 laboratory studies and conclude that DAT provides 'surprisingly

Table VI. Vuong likelihood ratio test (*p*-value) of selected pairs of a decision theory and a stochastic choice model (data from *Deal or No Deal UK*). A significantly positive (negative) value indicates that a row (column) pair is closer to the true data-generating process than a column (row) pair. Akaike Information Criterion is used to adjust for a smaller number of parameters in YDM

Stochastic choice model	→	Tremble model		Fechner model, homoscedastic errors		Fechner model, heteroscedastic errors		Fechner model, heteroscedastic truncated errors		Random utility model	
		EUT+EP	RT (SSB)	DAT	EUT+EP	RDU	YDM	RT (SSB)	EUT+EP	RT (SSB)	RDU
Tremble model	EUT+EP	—	4.1644 (0.0000)	-4.1141 (0.0000)	1.0995 (0.1358)	-1.0317 (0.1511)	-1.0036 (0.1578)	-6.0163 (0.0000)	-7.6664 (0.0000)	-2.3433 (0.0096)	-2.2859 (0.0111)
	RDU	-4.1644 (0.0000)	—	-11.002 (0.0000)	-2.6935 (0.0035)	-7.9213 (0.0000)	-8.2757 (0.0000)	-13.245 (0.0000)	-12.771 (0.0000)	-8.5367 (0.0000)	-8.2989 (0.0000)
Fechner model, homoscedastic	DAT	4.1141 (0.0000)	11.002 (0.0000)	—	4.1851 (0.0000)	2.8659 (0.0021)	3.0416 (0.0012)	-4.8405 (0.0000)	-3.7799 (0.0001)	1.3406 (0.0900)	1.4925 (0.0678)
	RDU	-1.0995 (0.1358)	2.6935 (0.0035)	-4.1851 (0.0000)	—	-2.144 (0.0160)	-2.0842 (0.0186)	-7.4486 (0.0000)	-6.8436 (0.0000)	-3.3964 (0.0003)	-3.3716 (0.0004)
Fechner model, heteroscedastic	RDU	1.0317 (0.1511)	7.9213 (0.0000)	-2.8659 (0.0021)	2.144 (0.0160)	—	0.5608 (0.2875)	-6.3481 (0.0000)	-4.7089 (0.0000)	-1.9082 (0.0282)	-1.7304 (0.0418)
	YDM	1.0036 (0.1578)	8.2757 (0.0000)	-3.0416 (0.0012)	2.0842 (0.0186)	-0.5608 (0.2875)	—	-6.7665 (0.0000)	-4.9721 (0.0000)	-1.9339 (0.0266)	-1.7724 (0.0382)
Fechner, heteroscedastic truncated	RDU	6.0163 (0.0000)	13.245 (0.0000)	4.8405 (0.0000)	7.4486 (0.0000)	6.3481 (0.0000)	6.7665 (0.0000)	—	1.8629 (0.0312)	4.3615 (0.0000)	4.541 (0.0000)
	EUT+EP	7.6664 (0.0000)	12.771 (0.0000)	3.7799 (0.0001)	6.8436 (0.0000)	4.7089 (0.0000)	4.9721 (0.0000)	-1.8629 (0.0312)	—	3.1862 (0.0007)	3.3938 (0.0003)
Random utility model	RT (SSB)	2.3433 (0.0096)	8.5367 (0.0000)	-1.3406 (0.0900)	3.3964 (0.0003)	1.9082 (0.0282)	1.9339 (0.0266)	-4.3615 (0.0000)	-3.1862 (0.0007)	—	0.7674 (0.2214)
	RDU	2.2859 (0.0111)	8.2989 (0.0000)	-1.4925 (0.0678)	3.3716 (0.0004)	1.7304 (0.0418)	1.7724 (0.0382)	-4.541 (0.0000)	-3.3938 (0.0003)	-0.7674 (0.2214)	—

good fit' when decision theories are estimated in a strong utility model, which is equivalent to our Fechner model of homoscedastic random errors.

EUT with CRRA utility function embedded in a Fechner model of heteroscedastic truncated errors fits the data significantly better than RDU embedded in a Fechner model of homoscedastic errors. This confirms the finding of Blavatsky (2007), who re-examines the data from 10 laboratory studies and concludes that EUT embedded in a Fechner model with heteroscedastic truncated errors generally fit the data at least as good as rank-dependent theories embedded in a Fechner model with homoscedastic errors.

## 6. CONCLUSION

Individual decisions under risk are often inconsistent and cannot be described by a deterministic decision theory on its own. Thus such decisions are usually described by a deterministic theory embedded in a model of stochastic choice. Comparisons between different theories all embedded in the same model of stochastic choice are very popular in an empirical research (e.g., Hey and Orme, 1994). A less common approach is to compare the goodness of fit of different models all combined with the same decision theory. In this paper we look at the bigger picture by comparing different decision theories embedded in different models of stochastic choice. The results of such comparison are quite striking.

The estimated parameters of the same decision theory vary significantly when the theory is embedded into different models of stochastic choice. Thus estimates of decision theories should be regarded as conditional on the selected model of stochastic choice. Moreover, various assumptions about individual risk attitudes (e.g., regret or disappointment aversion) may be satisfied in one model of stochastic choice and rejected in another. Therefore, when a researcher exogenously picks a model of stochastic choice, this has a profound effect on the estimated parameters of decision theories, which ultimately affects the drawn conclusions about their descriptive validity.

If a model of stochastic choice is selected endogenously, decision theories generally provide the best description of observed decisions when embedded in a Fechner model with heteroscedastic errors drawn from a truncated distribution (e.g., Blavatsky, 2007). In this model, the variance of errors is higher for lotteries with a wider range of possible prizes. Distribution of errors is truncated so that an individual always rejects sure amounts below the lowest possible prize in a risky lottery and always accepts amounts above the highest possible prize.

The decision theory that provides the best description of the data generally depends on the selected model of stochastic choice. For example, researchers who use a tremble model (e.g., Harless and Camerer, 1994) would conclude that EUT with expo-power utility function yields the best goodness of fit. Researchers who use a Fechner model of homoscedastic errors (e.g., Hey and Orme, 1994; Camerer and Ho, 1994) would claim that the most accurate description of the data is given by DAT. Last but not least, researchers who use a Fechner model of heteroscedastic truncated errors would conclude that the best decision theory is either EUT with expo-power utility function or RT (SBB).

To draw a bottom line, an accurate description of decisions under risk relies on two equally important components: a model of stochastic choice and a decision theory. These two components are mutually dependent and generally neither can be fixed in an exogenous manner. For simple binary choices between a risky lottery and a sure amount, which are analyzed in this paper,

the appropriate model of stochastic choice appears to be a Fechner model with heteroscedastic truncated errors or a random utility model.

#### ACKNOWLEDGEMENTS

We are grateful to Steffen Andersen, Christian Ewerhart, Glenn Harrison, Wolfgang Köhler, Morten Lau, Ulrich Schmidt, Francesco Trebbi, Peter Wakker, Nathaniel Wilcox, Fabrizio Zilibotti, two anonymous referees and editor Herman van Dijk for their insightful comments. We also thank participants of research seminars at the University of Zurich (7 December 2006), Max Planck Institute for Research on Collective Goods in Bonn (24 October 2006), the University of Bonn (26 September 2006), and the University of Innsbruck (4 May 2006), the participants of the Economic Science Association European Meeting in Nottingham, UK (9 September 2006) and the 12th International Conference on the Foundation and Application of Utility, Risk and Decision Theory in Rome, Italy (24 June 2006). Pavlo Blavatskyy acknowledges financial support from the Fund for Support of Academic Development at the University of Zurich.

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## APPENDIX

**Parametric functional forms of decision theories**

In contrast to the small number of models of stochastic choice, the number of decision theories reaches well into double-digit figures (Starmer, 2000). In this paper we selected only those decision theories that have a parsimonious parametric form. Specifically, we require that the number of parameters in every decision theory does not exceed two. Nearly all popular decision theories fulfill this requirement and they are briefly described below. However, inevitably, some decision theories are left out. In particular, we do not estimate weighted utility theory (e.g. Chew and McCrimmon, 1979; Chew, 1983) and (cumulative) prospect theory (e.g. Tversky and Kahneman, 1992).

With respect to cumulative prospect theory, it should be noted that we do not really exclude this theory from our analysis. We estimate a rank-dependent utility theory (Quiggin, 1981), which is equivalent to cumulative prospect theory with the additional assumption that all outcomes are located above the reference point, i.e. all outcomes are gains. Such an assumption is plausible in the context of our dataset, where individuals can only win quite significant cash prizes (and they cannot go home with less money).

**Risk neutrality (RN)**

Risk neutrality is the simplest decision theory. A risk-neutral individual always prefers the lottery with the highest expected value of possible outcomes. Let  $L(x_1, p_1; \dots; x_n, p_n)$  be a risky lottery that delivers outcome  $x_i$  with a probability  $p_i$ , for all  $i \in \{1, \dots, n\}$ . Under risk neutrality, the utility of lottery  $L$  is given by  $\sum_{i=1}^n p_i x_i$ . There are no free parameters to be estimated for this decision theory, i.e. vector  $\theta$  is the empty set.

**Expected utility theory (EUT)**

According to EUT, an individual evaluates monetary outcomes by means of a subjective utility function and chooses the lottery with the highest expected utility of possible outcomes. Formally, the utility of lottery  $L(x_1, p_1; \dots; x_n, p_n)$  is given by  $\sum_{i=1}^n p_i u(x_i)$ , where  $u: \mathbb{R} \rightarrow \mathbb{R}$  is a (Bernoulli) utility function over money. We will estimate expected utility theory with two utility functions: constant relative risk aversion (CRRA) and expo-power (EP). CRRA utility function is given by  $u(x) = x^{1-r}/(1-r)$ , when  $r \neq 1$ , and  $u(x) = \log(x)$ , when  $r = 1$ . Note that an individual with CRRA utility function is risk-seeking when  $r < 0$  and risk-averse when  $r > 0$ . When we restrict  $r = 0$ , EUT with CRRA utility function coincides with RN. For EUT with CRRA utility function, vector  $\theta$  consists only of one element: a coefficient of relative risk aversion  $r$ .

EP utility function is given by  $u(x) = \frac{1 - e^{-\alpha x^{1-r}/(1-r)}}{\alpha}$ , where  $\alpha \neq 0$  (e.g. Abdellaoui *et al.*, 2007). When  $r = 0$ , EP utility function exhibits a constant coefficient of absolute risk aversion, e.g.  $-u''(x)/u'(x) = \alpha$ . In the limiting case when  $\alpha \rightarrow 0$ , EP utility function exhibits a constant coefficient of relative risk aversion, i.e.  $\lim_{\alpha \rightarrow 0} -\frac{xu''(x)}{u'(x)} = r$ . However, since EP utility function is not defined when  $\alpha = 0$ , it does not nest EUT with CRRA utility function or RN as special cases. For EUT with EP utility function, vector  $\theta$  is given by  $\theta = [r, \alpha]$ . When estimating EUT

with EP utility function embedded into a random utility model, we assume that coefficient  $r$  is stochastic and coefficient  $\alpha$  is deterministic.

### Regret theory (RT) and skew-symmetric bilinear utility theory (SSB)

According to skew-symmetric bilinear utility theory (e.g. Fishburn, 1983), an individual chooses a risky lottery  $L(x_1, p_1; \dots; x_n, p_n)$  over a sure monetary amount  $O$  if  $\Psi(L, O) \equiv \sum_{i=1}^n p_i \psi(x_i, O) \geq 0$ , where  $\psi: \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$  is a skew-symmetric function, i.e.  $\psi(x, O) = -\psi(O, x)$ . SSB coincides with regret theory (e.g. Loomes *et al.*, 1992) if  $\psi(\cdot, \cdot)$  is convex (the assumption of regret aversion). We will estimate RT (SSB) with function

$$\psi(x, O) = \begin{cases} (x^{1-r}/(1-r) - O^{1-r}/(1-r))^\delta, & x \geq O \\ -(O^{1-r}/(1-r) - x^{1-r}/(1-r))^\delta, & x < O \end{cases} \quad (13)$$

for  $r \neq 1$ ,  $\delta > 0$  and function

$$\psi(x, O) = \begin{cases} (\log(x/O))^\delta, & x \geq O \\ -(\log(O/x))^\delta, & x < O \end{cases} \quad (14)$$

for  $r = 1$ ,  $\delta > 0$ . This function satisfies the assumption of regret aversion when  $\delta > 1$ .

When we restrict  $\delta = 1$ , RT (SSB) coincides with EUT with CRRA utility function. Interestingly, when we restrict  $r = 0$ , RT (SSB) becomes a restricted version of (cumulative) prospect theory with a linear probability weighting function, no loss aversion and a reference point equal to a sure monetary amount. Post *et al.* (2008) argued that a sure monetary amount is a 'natural' reference point in binary choice problems where an individual has to decide between a risky lottery and an amount for certain. Finally, when we restrict  $\delta = 1$  and  $r = 0$ , RT (SSB) coincides with RN.

Log-likelihood functions for RT (SSB) embedded into different models of stochastic choice are the same as the log-likelihood functions described in Section 2 if we replace  $u(L_i, \theta) - u(O_i, \theta)$  with  $\Psi(L_i, O_i, \theta)$  and  $u(\bar{x}_i, \theta) - u(x_i, \theta)$  with  $\psi(\bar{x}_i, x_i, \theta)$ . For RT (SSB), parameter vector  $\theta$  has two elements:  $\theta = [r, \delta]$ . When estimating RT (SSB) embedded into a random utility model, we assume that coefficient  $r$  is stochastic and coefficient  $\delta$  is deterministic.

### Yaari's dual model (YDM)

According to Yaari's dual model, the utility of a risky lottery,  $L(x_1, p_1; \dots; x_n, p_n)$ ,  $x_1 > x_2 > \dots > x_n$ , is given by  $\sum_{i=1}^n \left[ w \left( \sum_{j=1}^i p_j \right) - w \left( \sum_{j=1}^{i-1} p_j \right) \right] \cdot x_i$ , where  $w: [0, 1] \rightarrow [0, 1]$  is a probability weighting function (e.g. Yaari, 1987). The probability weighting function is strictly increasing and  $w(0) = 0$ ,  $w(1) = 1$ . We will estimate Yaari's dual model with probability weighting function  $w(p) = p^\gamma / (p^\gamma + (1-p)^\gamma)^{1/\gamma}$ . Note that when  $\gamma < 1$ , this probability weighing function has a typical inverse S-shape; i.e. small probabilities ( $p < 1/3$ ) of extreme outcomes are overweighted and medium and large probabilities ( $p > 1/3$ ) are underweighted (compared to their objective values). When  $\gamma > 1$ , the probability weighting function has atypical S-shape and small probabilities of extreme outcomes are underweighted and medium and large probabilities are overweighted.

When we restrict  $\gamma = 1$ , YDM coincides with RN. For YDM, the vector  $\theta$  consists only of one element: the coefficient of the probability weighting function  $\gamma$ .

### Rank-dependent utility theory (RDU)

According to rank-dependent utility theory, the utility of a risky lottery  $L(x_1, p_1; \dots; x_n, p_n)$ ,  $x_1 > x_2 > \dots > x_n$ , is given by  $\sum_{i=1}^n \left[ w \left( \sum_{j=1}^i p_j \right) - w \left( \sum_{j=1}^{i-1} p_j \right) \right] \cdot u(x_i)$ , where  $w: [0, 1] \rightarrow [0, 1]$  is a probability weighting function and  $u: \mathbb{R} \rightarrow \mathbb{R}$  is a utility function (e.g. Quiggin, 1981). If all lottery outcomes  $x_1 > x_2 > \dots > x_n$  are above the reference point of an individual, the prediction of RDU is identical to the prediction of cumulative prospect theory (e.g. Tversky and Kahneman, 1992). We will estimate RDU with the probability weighting function  $w(p) = p^\gamma / (p^\gamma + (1-p)^\gamma)^{1/\gamma}$  and CRRA utility function  $u(x) = x^{1-r} / (1-r)$ , when  $r \neq 1$ , and  $u(x) = \log(x)$ , when  $r = 1$ .<sup>6</sup>

When we restrict  $\gamma = 1$ , RDU coincides with EUT with CRRA utility function. When we restrict  $r = 0$ , RDU coincides with YDM. Finally, when we restrict  $\gamma = 1$  and  $r = 0$ , RDU coincides with RN (a total of two restrictions). For RDU, vector  $\theta$  has two elements:  $\theta = [r, \gamma]$ . When estimating RDU embedded into a random utility model, we assume that coefficient  $r$  is stochastic and coefficient  $\gamma$  is deterministic.

### Disappointment aversion theory (DAT)

According to DAT, an individual experiences disappointment when a realized outcome of a lottery is below its certainty equivalent (e.g. Gul, 1991). Formally, the utility of a lottery

$$L(x_1, p_1; \dots; x_n, p_n), x_1 > x_2 > \dots > x_n, \text{ is (implicitly) defined by } \frac{1}{1 + \beta} \frac{\sum_{i=1}^{n-m} p_i u(x_i)}{\sum_{i=n-m+1}^n p_i} + \frac{1 + \beta}{1 + \beta \sum_{i=n-m+1}^n p_i} \sum_{i=n-m+1}^n p_i u(x_i), \text{ where } m \in \{1, \dots, n-1\} \text{ is the number of disappointing}$$

outcomes in  $L$  and  $\beta > -1$  is a parameter that captures disappointment preferences ( $\beta > 0$  indicates disappointment aversion).

We estimate DAT with CRRA utility function  $u(x) = x^{1-r} / (1-r)$ , when  $r \neq 1$ , and  $u(x) = \log(x)$ , when  $r = 1$ . When we restrict  $\beta = 0$ , DAT becomes EUT with CRRA utility function. When we restrict  $\beta = 0$  and  $r = 0$ , DAT coincides with RN. For DAT, vector  $\theta = [r, \beta]$ . When estimating DAT embedded into a random utility model, we assume that coefficient  $r$  is stochastic and coefficient  $\beta$  is deterministic.

<sup>6</sup> Note that RDU is often estimated with power utility function  $u(x) = x^\alpha$ . By setting  $\alpha = 1 - r$  we can immediately relate our estimates of coefficient  $r$  to the estimates of power coefficient  $\alpha$ , which are often reported in other studies.